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Review report on Ph.D. Thesis

Thesis Title: "*Synchronization of systems with hidden attractors*" by Gokul P. M., Lodz University of Technology, Lodz, Poland

Synchronization, which means the adjustment of rhythms of self-sustained oscillators due to their interaction, is found in almost all branch of science. It has become a field of intense interest and hence extensively studied in the last decades. In this thesis it is the topic of study. The concept of synchronization of systems with complex dynamics has been reviewed properly here.

Recently, rare and hidden attractors have been studied with great interest in dynamical systems. Understanding and locating such attractors, particularly in multistability region, are very important, particularly for applications. Different types of scenarios, including multistable systems with hidden and rare attractors have been studied, showing that observed phenomena can be found in variety of models, especially fundamental ones from the field of mechanical systems. Some important properties of such attractors are uncovered and discussed in this thesis

In the first work the phenomenon of synchronization is observed in different systems having hidden attractors. The different type of coupling function is also explored. It is found that, depending on the strength of coupling between the systems, not only the synchronization gets affected but also the final synchronized state is affected. Different initial conditions are used to understand the synchronization. The different type of transitions is found when couplings are done in different set of variables. These transitions are interesting which advance to the understating of multistability.

The concept of synchronization is extensively studied analytically and numerically in simple phase oscillators, called as Kuramoto oscillators. In second work this system is considered to see the effect of different type of couplings, particularly attractive and repulsive couplings. It is observed that in the presence of a strong attractively coupled oscillators, a system of repulsively coupled Kuramoto oscillators reach synchronization faster than a system of attractively coupled oscillators. There are different regions of synchronization of the internal oscillators, characterized by their frequencies. It is also found that the described phenomenon does not depend on the initial frequency distribution by repeating the numerical simulations for a total of three different initial internal frequency distributions: namely the Lorentzian, Gaussian and uniform ones. The results

show that width of the distribution plays important role. Numerical and analytical results are presented.

It is known that different types of synchronization like complete synchronization, anti-synchronization, lag synchronization, mixed synchronization, could be observed in coupled dynamical systems depending on type of coupling functions and its strengths. Some of these different types of synchronization are presented in third work. Here it is demonstrated that the occurrence of mixed synchronization is possible by designing a coupling strategy in cycling chaotic systems. Eight possible types of mixed synchronization are observed. The system can settle for any one of the eight cycling states depending only on the initial conditions. Its possible applications in neural system are explored.

Most of the conventional chaotic systems have a countable number of equilibrium points. However a few unusual systems with infinite number of equilibria have been discovered recently. In 4^{th} work anti-synchronization between two systems with infinite equilibria, namely Master and Slave systems, are investigated. Anti-synchronization is an interesting scheme in which two systems can be synchronized in amplitude, but with opposite signs. The different measure/techniques e.g. using phase portraits, Poincar´e map, bifurcation diagram, and Lyapunov exponents, are considered to investigate the system's dynamics. The chaos and coexisting attractors in such a system are discussed. An adaptive control for anti–synchronization of chaotic systems with infinite equilibria is proposed here. The fractional–order form of the system with infinite equilibria is also considered.

Recommendation: This thesis deals with the understanding of synchronization which is studied in different systems as well as with different coupling functions. Results are interesting. Writing and presentation are good. The research works look original and interesting – these are confirmed by its publications in important journals. I recommend its acceptance for the award of Ph.D. degree.

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